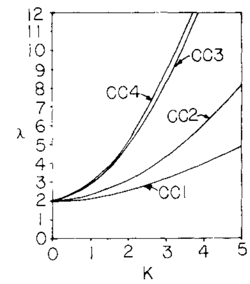

Fig. 3 **Results for the fundamental mode of clamped panels ($m = 1$).**

which has been studied extensively.[3-7] The results for the other cases agree with the approximate solutions of Matsuzaki.[7]

The results for clamped panels are shown in Fig. 3. The CC4 case has been studied previously[4-6] by approximate methods. The remaining three solutions are believed to be new, and they permit a comparison to be made with the simply supported cases.

As can be seen from Figs. 2 and 3, the effect of tangential boundary conditions is similar for simply supported and clamped edges. Furthermore, the trends indicated are similar to those found in the compressive buckling cases by Rehfield and Hallauer. We conclude, therefore, that the fundamental vibration frequency is sensitive to rotational restraint ($w_{,yy} = 0$ or $w_{,y} = 0$), circumferential restraint ($\sigma_y{}^o = 0$ or $v = 0$), and longitudinal restraint ($\tau_{xy}{}^o = 0$ or $u = 0$) of the straight edges, in decreasing order of influence.

References

[1] Rehfield, L. W. and Hallauer, W. L., Jr., "Edge Restraint Effect on Buckling of Compressed Curved Panels," *AIAA Journal*, Vol. 6, No. 1, Jan. 1968, pp. 187–188.

[2] Singer, J., Meer, A., and Baruch, M., "Buckling of Cylindrical Panels Under Lateral Pressure," SR-6, March 1968, Technion—Israel Institute of Technology, Haifa, Israel.

[3] Forsberg, K., "Influence of Boundary Conditions on the Modal Characteristics of Thin Cylindrical Shells," *AIAA Journal*, Vol. 2, No. 12, Dec. 1964, pp. 2150–2157.

[4] Palmer, P. J., "The Natural Frequency of Vibration of Curved Rectangular Plates," *Aeronautical Quarterly*, Vol. 5, 1954, pp. 101–110.

[5] Sewall, J. L., "Vibration Analysis of Cylindrically Curved Panels with Simply Supported or Clamped Edges and Comparison with Some Experiments," TN D-3791, 1967, NASA.

[6] Webster, J. J., "Free Vibrations of Rectangular Curved Panels," *International Journal of Mechanical Sciences*, Vol. 10, No. 7, July 1968, pp. 571–582.

[7] Matsuzaki, Y., "Influence of the In-Plane Boundary Conditions on the Natural Frequency of Cylindrically Curved Panels with Simply Supported Edges," *International Journal of Solids and Structures*, Vol. 7, 1971, pp. 1555–1571.

[8] Donnell, L. H., "Stability of Thin-Walled Tubes Under Torsion," Rept. 479, 1933, NACA.

[9] Reissner, E., "On Transverse Vibrations of Thin, Shallow Elastic Shells," *Quarterly of Applied Mathematics*, Vol. 13, No. 2, July 1955, pp. 169–176.

Pressure Sources for a Wave Model of Jet Noise

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THE structure of the turbulent jet has in recent years been studied extensively in connection with noise generation. It was found that the structure of the turbulent mixing region is organized fairly orderly and behaves like a wave train similar to the hydrodynamic stability waves propagating in a shear flow. Among many other experimental observations, Mollø-Christensen[1] and Crow and Champagne[2] gave the most interesting accounts of this wavelike phenomenon. A detailed measurement of the spatial growth and decay of the pressure wave in the turbulent mixing layer of a low-speed jet has recently been reported by Chan.[3] This regular structure in the turbulent mixing region has also led to some revision of the turbulence models used in the jet noise computations. In Michalke's model[4,5] the wave character of the flow is incorporated explicitly. The source function is first resolved into Fourier components in the azimuth angle and for each frequency the source component is assumed to be wavelike. The far field solutions thus obtained generate all essential features of the sound pressure distributions observed experimentally. In Michalke's work, the exact form of the source components are not yet known and in order to illustrate his solutions, several forms of source distributions are assumed. The source term in Michalke's wave model, however, can be readily written in terms of the fluctuation pressure according to the dilation theory of Ribner.[6] The experimental results obtained by Chan[3] for the pressure wave development in the mixing region can then be used to evaluate the proper form for the source function.

From the dilation theory, the source term, q, can be written as

$$q = (1/a_0{}^2)\,\partial^2 p^{(o)}/\partial t^2 \tag{1}$$

where a_0 is the speed of sound of the ambient air and $p^{(o)}$ the pressure fluctuation in the jet. The single frequency component of the source term can be obtained by Fourier transform of Eq. (1). Thus the mth azimuth Fourier component for a single frequency can be written as

$$Q_{m\omega}(x, r) = -k^2 P_{m\omega}{}^{(o)}(x, r) \tag{2}$$

where $k = \omega/a_0$ is the wave number of the sound wave at the frequency ω. Assuming the pressure source $P_{m\omega}{}^{(o)}(x, r)$ is separable in the independent variable x and r

$$P_{m\omega}{}^{(o)}(x, r) = F(x)G(r) \tag{3}$$

where $F(x)$ and $G(r)$ are, respectively, the longitudinal and lateral amplitude distributions of the pressure waves inside the jet. Then it can be shown that the far field sound pressure, $P_{m\omega}$, is proportional to the source integrals[4]

$$P_{m\omega} \propto \frac{R^2 L}{2} \int_0^1 \int_0^1 \hat{F}(\bar{x})\hat{G}(\bar{r})\bar{r}\, d\bar{r}\, d\bar{x} \cdot I_r \cdot I_x \tag{4}$$

where

$$I_r = \frac{\int_0^1 \hat{G}(\bar{r})\bar{r} J_m(kR \sin\theta \bar{r})\, d\bar{r}}{\int_0^1 \hat{G}(\bar{r})\bar{r}\, d\bar{r}}$$

$$I_x = \frac{\int_0^1 \hat{F}(\bar{x}) \exp\left[i\alpha L(1 - M_c \cos\theta)\bar{x}\right] d\bar{x}}{\int_0^1 \hat{F}(\bar{x})\, d\bar{x}}$$

and

$$\hat{F} = F/F_{\max}, \quad \hat{G} = G/G_{\max}, \quad \bar{x} = x/L, \quad \bar{r} = r/R$$

L and R are the extents of the source region in the x and r directions, respectively. θ is the angle in polar coordinates for the measuring point and Jm the Bessel function of order m. The convection Mach number is the ratio of the phase velocity c_{ph} of the pressure wave and the ambient sound speed a_0. The phase velocity of the pressure wave is given in Ref. 3.

The experimental data of Ref. 3 show that the longitudinal distributions of the amplitude of the pressure waves for different Strouhal numbers are very similar to each other when the data are plotted against a normalized scale of distance $St\, x/D$, where St is the Strouhal number of the jet based on the jet velocity U

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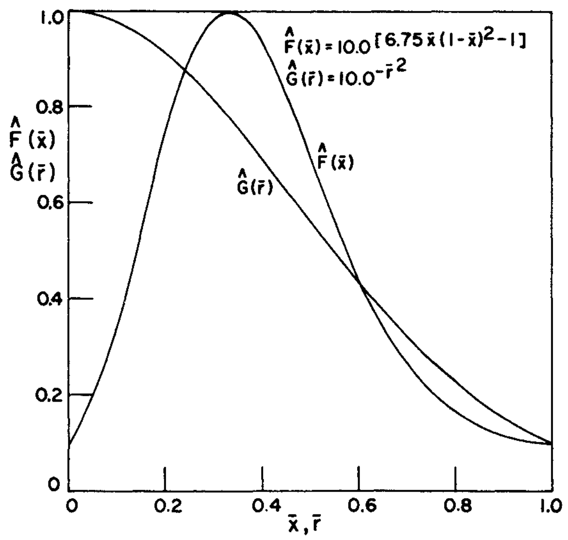

Fig. 1 Longitudinal and lateral distributions of pressure source in a turbulent jet.

and the nozzle diameter D. Thus the longitudinal source distribution can be adequately represented by the expression

$$\frac{\log p/p_1}{(\log p/p_i)_{\max}} = \frac{27}{4}\bar{x}(1-\bar{x})^2 \tag{5}$$

where the nondimensional distance $\bar{x}$ is now equal to $(St\,x/D)/l$, l is the extent of the source distribution in terms of the normalized distance and is approximately equal to 3.0 (Ref. 3). Thus the source function $\hat{F}(\bar{x})$ can be written as

$$\hat{F}(\bar{x}) = C_2^{[(27/4)\bar{x}(1-\bar{x})^2-1]} \tag{6}$$

with $C_2 = 10.0$ and $\bar{x}$ ranges from zero to unity.

The lateral source distributions can be represented by the expression

$$\frac{\log p/p_1}{(\log p/p_1)_{\max}} = 1-\bar{r}^2 \tag{7}$$

Since the jet spreads out laterally downstream from the nozzle, the lateral dimension of the source, R, is not a constant. However, in evaluating the integral a mean value of R can be taken and the source function can thus be written as

$$\hat{G}(\bar{r}) = C_2^{-\bar{r}^2} \tag{8}$$

The functions $\hat{F}(\bar{x})$ and $\hat{G}(\bar{r})$ are shown graphically in Fig. 1.

The source integrals, I_x and I_r, for the symmetric case ($m = 0$) are evaluated numerically for the source functions given in Eqs. (6) and (8) and the results are shown in Fig. 2. Figure 3 shows the squares of the amplitude of the source integrals, the

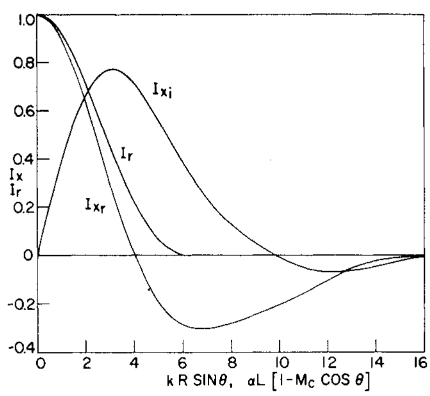

Fig. 2 Real and imaginary parts of the longitudinal source integral, I_{x_r}, I_{x_i} vs $\alpha L[1-M_c\cos\theta]$ and the lateral source integral I_r vs $kR\sin\theta$.

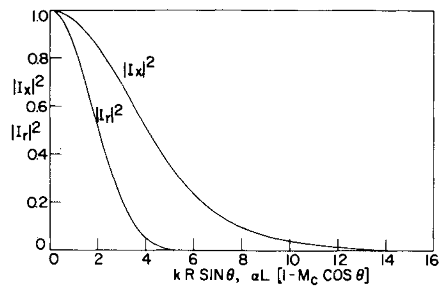

Fig. 3 Squares of the amplitude of the longitudinal source integral $|I_x|^2$ vs $\alpha L[1-M_c\cos\theta]$ and the lateral source integral $|I_r|^2$ vs $kR\sin\theta$.

product of which is proportional to the far field sound intensity. It is interesting to point out that model (IV) for the longitudinal source and model (b) for the lateral distribution assumed by Michalke (see Figs. 1 and 3 of Ref. 4) are closely similar to the present results derived from experimental data.

The present forms of the source integrals can now be used for the calculation of the axisymmetric component of the far field sound intensity based on the wave-model formulation. Because the axisymmetric component of the source term radiates much more sound than the asymmetric components, the calculated results based on the present source integrals will be a good approximation to the total sound intensity radiated by the turbulent jet.

References

[1] Mollφ-Christensen, E., "Jet Noise and Shear Flow Instability seen from an Experimenter's Viewpoint," *Journal of Applied Mechanics*, Vol. 34, 1967, pp. 1–7.

[2] Crow, S. C. and Champagne, F. H., "Orderly Structure in Jet Turbulence," *Journal of Fluid Mechanics*, Vol. 48, Pt. 3, 1971, pp. 547–591.

[3] Chan, Y. Y., "Spatial Waves in Turbulent Jets," *The Physics of Fluids*, Vol. 17, No. 1, Jan. 1974.

[4] Michalke, A., "A Wave Model for Sound Generation in Circular Jets," *Deutsche Luft-und Raumfahrt*, FB 70-57, 1970.

[5] Michalke, A., "An Expansion Scheme for the Noise from Circular Jets," *Zeitschrift für Flugwissenschaften*, Vol. 20, Heft 6, 1972, pp. 229–237.

[6] Ribner, H. S., "Aerodynamic Sound from Fluid Dilatations," UTIA Rept. 86, AFOSRTN 3430, July 1962, Inst. of Aerophysics, Univ. of Toronto, Toronto, Ontario, Canada.

Attachment-Line Flow on an Infinite Swept Wing

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Introduction

RECENTLY the present author developed a general method for calculating three-dimensional incompressible laminar, transitional, and turbulent boundary layers and investigated its accuracy for infinite swept wings.[1] The method uses the eddy-viscosity concept to model the Reynolds shear-stress term. The calculated results agreed well with experiment and with those results obtained by Bradshaw's method. In this Note, we shall

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